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Reconciliation of CDM abundance and $\mu \to e\gamma$ in a radiative seesaw model

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Abstract

We reexamine relic abundance of a singlet fermion as a CDM candidate, which contributes to the neutrino mass generation through radiative seesaw mechanism. We search solutions for Yukawa couplings and the mass spectrum of relevant fields to explain neutrino oscillation data. For such solutions, we show that an abundance of a lightest singlet fermion can be consistent with WMAP data without conflicting with both bounds of $\mu \to e\gamma$ and $\tau \to \mu\gamma$. This reconciliation does not need any modification of the original radiative seesaw model other than by specifying flavor structure of Yukawa couplings and taking account of coannihilation effects.

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1 Introduction

Observation of nonzero neutrino masses [1] and the existence of dark matter [2] gives big impact on the study of physics beyond the standard model (SM). Elucidation of the origin of neutrino masses and dark matter is now one of the biggest subjects in this field. Evidence for small neutrino masses suggests some additional structure in the SM as ones required in the seesaw mechanism [3]. On the other hand, in order to explain the abundance of dark matter, we need some symmetry to guarantee the stability of a dark matter candidate. The most famous example of such symmetry is R parity in supersymmetric models. Even in non-supersymmetric models in general, if Z_2 symmetry is required by some natural reason, it can play the same role as R parity as long as the ordinary SM fields have its even charge. From this point of view, there appears a very interesting idea that neutrino masses may be intimately related to the existence of cold dark matter.

If we consider that neutrino masses are generated radiatively under the assumption that a model has Z_2 symmetry whose existence is justified to forbid tree-level Dirac neutrino masses [4], this symmetry can guarantee the stability of a Z_2 odd neutral particle which may be cold dark matter (CDM). In this direction the relation between neutrino masses and CDM has been studied in various articles [5, 6, 7, 8]. Since such scenarios require the introduction of new particles and interactions in the SM, however, it can induce dangerous effects in various phenomena as usual. In fact, it is suggested that contradiction could appear between the strength of Yukawa couplings to satisfy the required relic abundance of CDM and the bound for $\mu \to e\gamma$ as long as a singlet fermion is considered as a CDM candidate [6]. Some attempts have been proposed to overcome this fault by modifying the model [9].

In this paper we fix our target on a minimal model in [4], which can generate neutrino mass radiatively. And we also confine our study into the case that a CDM candidate is one of the singlet fermions. We reanalyze whether its CDM abundance can be consistent with lepton flavor violating processes only by specifying detailed structure of the neutrino mass matrix and taking account of coannihilation effects. Our result will show that the CDM abundance can be consistent with lepton flavor violating processes even within the simplest radiative seesaw framework.

The paper is organized as follows. In the next section we define our model briefly

and discuss neutrino masses and mixing for the explanation of neutrino oscillation data based on this model. In section 3 we show our result on both relic abundance of the CDM candidate and constraints from the lepton flavor violating processes. We summarize the paper in section 4.

2 Neutrino mass due to radiative effects

We consider a model which is an extension of the SM with an additional $SU(2)_L$ doublet scalar η and three gauge singlet right-handed fermions N_k [4]. The model is also imposed by Z_2 symmetry to forbid tree-level Dirac masses for neutrinos. We assign odd charge of this Z_2 symmetry to all of these new fields, although Z_2 even charge is assigned to all of the SM contents.

Lagrangian relevant to N_k invariant under the SM gauge symmetry and this Z_2 symmetry are written as

$$\mathcal{L}_N = \left(i\overline{N_k}\gamma^\mu\partial_\mu P_R N_k\right) + \frac{1}{2}\left(M_k\overline{N_k^c}P_R N_k + M_k^*\overline{N_k}P_L N_k^c\right) - (h_{\alpha k}\overline{\ell_\alpha}\eta P_R N_k + \text{h.c.}), \quad (1)$$

where ℓ_α stands for a lepton doublet and a charged lepton mass matrix is assumed to be diagonalized. We note that N_k can have mass terms invariant under the imposed symmetry. For simplicity, these masses M_k and Yukawa couplings $h_{\alpha k}$ are assumed to be real in the following discussion.

Scalar doublets Φ and η have invariant scalar potential

$$\begin{aligned} V &= m_\Phi^2\Phi^\dagger\Phi + m_\eta^2\eta^\dagger\eta + \frac{1}{2}\lambda_1(\Phi^\dagger\Phi)^2 + \frac{1}{2}\lambda_2(\eta^\dagger\eta)^2 + \lambda_3(\Phi^\dagger\Phi)(\eta^\dagger\eta) + \lambda_4(\Phi^\dagger\eta)(\eta^\dagger\Phi) \\ &+ \frac{1}{2}\lambda_5\left[(\Phi^\dagger\eta)^2 + \text{h.c.}\right], \end{aligned} \quad (2)$$

where Φ is the ordinary SM Higgs doublet. If we assume that only Φ obtains a vacuum expectation value (VEV) such as $\langle\Phi^0\rangle = v$ but η obtains no VEV, neutrinos cannot have tree-level Dirac masses. However, neutrino masses can be generated radiatively through a one-loop diagram which has η^0 and N_k in internal lines. This radiative masses can be small as long as λ_5 is sufficiently small.[§] Since λ_5 is assumed to be very small, masses of real and imaginary parts of η^0 and also $\eta^\pm$ are considered to be degenerate and they can

[§]Since we can introduce a new U(1) symmetry in case of $\lambda_5 = 0$, the smallness of λ_5 may be considered as a natural assumption.

be written as $m_0^2 = m_\eta^2 + (\lambda_3 + \lambda_4)v^2$. Thus, in the following discussion we use this m_0^2 as the mass of η.

Radiatively generated neutrino masses are expressed by using the Yukawa couplings $h_{\alpha k}$ and three mass scales Λ_k as

$$(\mathcal{M}_\nu)_{\alpha\beta} = \sum_{k=1}^{3} h_{\alpha k} h_{\beta k} \Lambda_k, \tag{3}$$

where Λ_k is defined by

$$\Lambda_k = \frac{\lambda_5 v^2}{8\pi^2 M_k} I\left(\frac{M_k}{m_0}\right), \qquad I(x) = \frac{x^2}{1-x^2}\left(1 + \frac{x^2}{1-x^2}\ln x^2\right). \tag{4}$$

By using this mass matrix, we now consider how to explain neutrino oscillation data. Since it is known that neutrino oscillation data are well explained by using the Maki-Nakagawa-Sakata (MNS) matrix[¶]

$$U = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\frac{\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin\theta}{\sqrt{2}} & -\frac{\cos\theta}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{5}$$

we assume that the mass matrix (3) is diagonalized as $U^T \mathcal{M}_\nu U = \mathrm{diag}(m_1, m_2, m_3)$. Then, we find that the following diagonalization conditions should be satisfied:

$$\begin{aligned} &\sum_{k=1}^{3}\left(2h_{ek}^2 \sin 2\theta + 2\sqrt{2} h_{ek}(h_{\mu k} - h_{\tau k})\cos 2\theta - (h_{\tau k} - h_{\mu k})^2 \sin 2\theta\right) = 0, \\ &\sum_{k=1}^{3} h_{ek}\left(h_{\mu k} + h_{\tau k}\right) = 0, \qquad \sum_{k=1}^{3}\left(h_{\mu k} - h_{\tau k}\right)\left(h_{\mu k} + h_{\tau k}\right) = 0. \end{aligned} \tag{6}$$

The mass eigenvalues are expressed as

$$\begin{aligned} m_1 &= \sum_{k=1}^{3}\left(h_{ek}^2 \cos^2\theta + \frac{1}{\sqrt{2}}\sin 2\theta h_{ek}(h_{\tau k} - h_{\mu k}) + \frac{1}{2}\sin^2\theta\left(h_{\tau k} - h_{\mu k}\right)^2\right)\Lambda_k, \\ m_2 &= \sum_{k=1}^{3}\left(h_{ek}^2 \sin^2\theta - \frac{1}{\sqrt{2}}\sin 2\theta h_{ek}(h_{\tau k} - h_{\mu k}) + \frac{1}{2}\cos^2\theta\left(h_{\tau k} - h_{\mu k}\right)^2\right)\Lambda_k, \\ m_3 &= \sum_{k=1}^{3}\frac{1}{2}\left(h_{\tau k} + h_{\mu k}\right)^2 \Lambda_k. \end{aligned} \tag{7}$$

[¶]In this matrix $\sin\theta_{13}$ is assumed to be zero, for simplicity. Since $\sin\theta_{13}$ is expected to be very small, the present analysis is considered to be straightforwardly extended to the case with $\sin\theta_{13} \neq 0$.

Here, among various solutions for the conditions (6), we consider a simple solution such as

$$h_{ei} = 0, \quad h_{\mu i} = h_{\tau i}; \quad h_{ej} \neq 0, \quad h_{\mu j} = -h_{\tau j}, \tag{8}$$

where $i \neq j$ is assumed.[‖] This means either of i or j runs two values of $k = 1, 2, 3$ such as $i = 1, 2$ and $j = 3$, for example. By substituting this in the first condition in (6) we have

$$\tan\theta = -\frac{1}{\sqrt{2}}\frac{h_{ej}}{h_{\tau j}}. \tag{9}$$

The mass eigenvalues (7) are rewritten as

$$\begin{aligned} m_1 &= \left(h_{ej}\cos\theta + \sqrt{2}h_{\tau j}\sin\theta\right)^2 \Lambda_j = 0, \\ m_2 &= \left(h_{ej}\sin\theta - \sqrt{2}h_{\tau j}\cos\theta\right)^2 \Lambda_j = \frac{2h_{\tau j}^2}{\cos^2\theta}\Lambda_j, \\ m_3 &= 2h_{\tau i}^2\Lambda_i, \end{aligned} \tag{10}$$

where the summation for i and j should be understood. We use eq. (9) in the last equality for $m_{1,2}$.[**]

Now we impose phenomenological requirements on the model. If we recall that the MNS matrix is given by U defined in eq. (5), it is found that we can use $\sin^2\theta \simeq 0.33$ and $m_2^2 \simeq 7.66 \times 10^{-5}$ eV^2 suggested by the solar neutrino and KamLAND data and also $m_3^2 \simeq 2.46 \times 10^{-3}$ eV^2 suggested by the atmospheric neutrino and K2K data [1]. As this result, we obtain

$$h_{\tau j}^2\Lambda_j \simeq 2.9 \times 10^{-3}\ \text{eV}, \qquad h_{\tau i}^2\Lambda_i \simeq 2.5 \times 10^{-2}\ \text{eV}, \tag{11}$$

where the summation on i and j is abbreviated. Similar one-loop diagrams to the one for neutrino masses contribute to the lepton flavor violating processes like $\ell_a \to \ell_b\gamma$. It gives the most severe constraint on the model. Its branching ratio is estimated as [11]

$$Br(\ell_a \to \ell_b\gamma) = \frac{3\alpha}{64\pi(G_F m_0^2)^2}\left|\sum_{k=1}^{3} h_{\ell_a k}h_{\ell_b k}F_2\left(\frac{M_k}{m_0}\right)\right|^2, \tag{12}$$

where $F_2(x)$ is given by

$$F_2(x) = \frac{1 - 6x^2 + 3x^4 + 2x^6 - 6x^4\ln x^2}{6(1-x^2)^4}. \tag{13}$$

‖ If $h_{\mu k} = -h_{\tau k}$ is satisfied for all k, it is also a solution for the diagonalization conditions. However, such a solution cannot satisfy neutrino oscillation data and then we do not consider this case here.

** This type of neutrino mass hierarchy induced from the mass matrix (3) has been considered to analyze neutrino oscillation data in other context [10].

The present upper bounds for $Br(\mu \to e\gamma)$ and $Br(\tau \to \mu\gamma)$ are given as 1.2×10^{-11} [12] and 6.8×10^{-8} [13], respectively. If we use eq. (8), these constraints can be written as

$$\begin{aligned} &\left| h_{\tau j}^2 F_2\left(\frac{M_j}{m_0}\right)\right| < 9.8 \times 10^{-4}\left(\frac{m_0}{500\ \mathrm{GeV}}\right)^2, \\ &\left| h_{\tau i}^2 F_2\left(\frac{M_i}{m_0}\right) - h_{\tau j}^2 F_2\left(\frac{M_j}{m_0}\right)\right| < 7.3 \times 10^{-2}\left(\frac{m_0}{500\ \mathrm{GeV}}\right)^2. \end{aligned} \qquad (14)$$

If we discuss other phenomenological features of the model, we should analyze them under the conditions (11) and (14).

To proceed with the study of CDM abundance in the next section, it is convenient to classify possible cases for the relation between M_k and m_0. Since we assume N_1 to be a CDM candidate and we are interested in the effect of coannihilation, N_1 is considered to be almost degenerate with other Z_2 odd fields. In such a situation, physically distinctive cases may be classified as

$$\text{(i) } M_1 \lesssim M_2 < M_3,\ m_0, \quad \text{(ii) } M_1 \lesssim M_2,\ m_0 < M_3, \quad \text{(iii) } M_1 \lesssim m_0 < M_2,\ M_3.$$

Although there is no logical correlation between Yukawa couplings and masses of singlet fermions without introducing some symmetry, here we only assume that the masses of the corresponding singlet fermions are equal if Yukawa couplings are equal in eq. (8). We can identify important processes for determination of CDM abundance under this assumption. If we take $i = 1, 2$ and $j = 3$, two possible cases (i) and (ii) should be considered. In case (i) we need to take account of coannihilation between N_1 and N_2, in which only Yukawa couplings are relevant to this process. On the other hand, in case (ii) we take account of coannihilation among N_1, N_2 and η. Gauge interaction is expected to play an important role in this case. If we take $i = 1$ and $j = 2, 3$, the case (iii) with $M_2 = M_3$ is a target for the investigation. In this case coannihilation between N_1 and η is expected to play a crucial role. Both gauge and Yukawa interactions are relevant to this case. Although final states of coannihilation in the cases (ii) and (iii) can include antiproton, the case (i) can not include it but include only lepton pairs. This aspect makes the case (i) interesting in the relation to the PAMELA e^+ and $\bar{p}$ data [14], and also the ATIC/PPB-BETS $(e^+ + e^-)$ data [15, 16]. We will come back to this point later. In the next section we will confine our study to this case. Other cases will be discussed elsewhere.

3 Coannihilation of the CDM candidate

In this section we consider (co)annihilation of N_1 through Yukawa couplings in the case (i). For the estimation of the relic abundance of N_1, we follow the method given in [17], which is developed to take account of coannihilation effects. If we introduce the dimensionless parameter x as $x = M_1/T$, the decoupling temperature T_f of N_1 can be estimated by using effective cross section $\sigma_{\rm eff}$ and effective degrees of freedom $g_{\rm eff}$ as

$$x_f = \ln \frac{0.038 g_{\rm eff} m_{\rm pl} M_1 \langle \sigma_{\rm eff}|v_{\rm rel}|\rangle}{g_*^{1/2} x_f^{1/2}}, \tag{15}$$

where $v_{\rm rel}$ is the relative velocity of annihilating fields. $\sigma_{\rm eff}$ and $g_{\rm eff}$ are defined as

$$\begin{aligned}
\sigma_{\rm eff} &= \frac{g_{N_1}^2}{g_{\rm eff}^2}\sigma_{N_1N_1} + 2\frac{g_{N_1}g_{N_2}}{g_{\rm eff}^2}\sigma_{N_1N_2}(1+\Delta)^{3/2}e^{-\Delta x} + \frac{g_{N_2}^2}{g_{\rm eff}^2}\sigma_{N_2N_2}(1+\Delta)^3 e^{-2\Delta x}, \\
g_{\rm eff} &= g_{N_1} + g_{N_2}(1+\Delta)^{3/2}e^{-\Delta x},
\end{aligned} \tag{16}$$

where $m_{\rm pl} = 1.22 \times 10^{19}$ GeV and Δ is defined by $\Delta \equiv (M_2 - M_1)/M_1$. If we define $a_{\rm eff}$ and $b_{\rm eff}$ by $\sigma_{\rm eff}|v_{\rm rel}| = a_{\rm eff} + b_{\rm eff} v_{\rm rel}^2$, thermally averaged cross section can be written as $\langle \sigma_{\rm eff}|v_{\rm rel}|\rangle = a_{\rm eff} + 6b_{\rm eff}/x$. In the following analysis, $\Delta \simeq 0$ is assumed since we consider the case (i). Thus, if we use this decoupling temperature x_f, the relic abundance can be estimated by

$$\Omega h^2 = \frac{1.07 \times 10^9 x_f}{g_*^{1/2} m_{\rm pl}({\rm GeV})(a_{\rm eff} + 3b_{\rm eff}/x_f)}. \tag{17}$$

(Co)annihilation proceeds via t-channel exchange of η^0 and $\eta^\pm$ through Yukawa interactions. The final states are composed of only leptons μ, τ, ν_μ, ν_τ and their antiparticles. We note that antiproton is never produced. The (co)annililation cross section of N_{i_1} and N_{i_2} is estimated as

$$\begin{aligned}
\sigma_{N_{i_1}N_{i_2}}|v_{\rm rel}| &= \frac{1}{8\pi}\frac{M_1^2}{(M_1^2+m_0^2)^2}\left[1 + \frac{m_0^4 - 3m_0^2M_1^2 - M_1^4}{3(M_1^2+m_0^2)^2}v_{\rm rel}^2\right]\sum_{\alpha,\beta}(h_{\alpha i_1}h_{\beta i_2} - h_{\alpha i_2}h_{\beta i_1})^2 \\
&+ \frac{1}{12\pi}\frac{M_1^2(M_1^4+m_0^4)}{(M_1^2+m_0^2)^4}v_{\rm rel}^2\sum_{\alpha,\beta}h_{\alpha i_1}h_{\alpha i_2}h_{\beta i_1}h_{\beta i_2},
\end{aligned} \tag{18}$$

where i_1, i_2 should be considered as 1 or 2. As can be seen from this expression with $i_1 = i_2 = 1$, the annihilation of N_1 occurs only through a p-wave channel. On the other hand, coannihilation defined by $i_1 \neq i_2$ can have s-wave contributions in general.

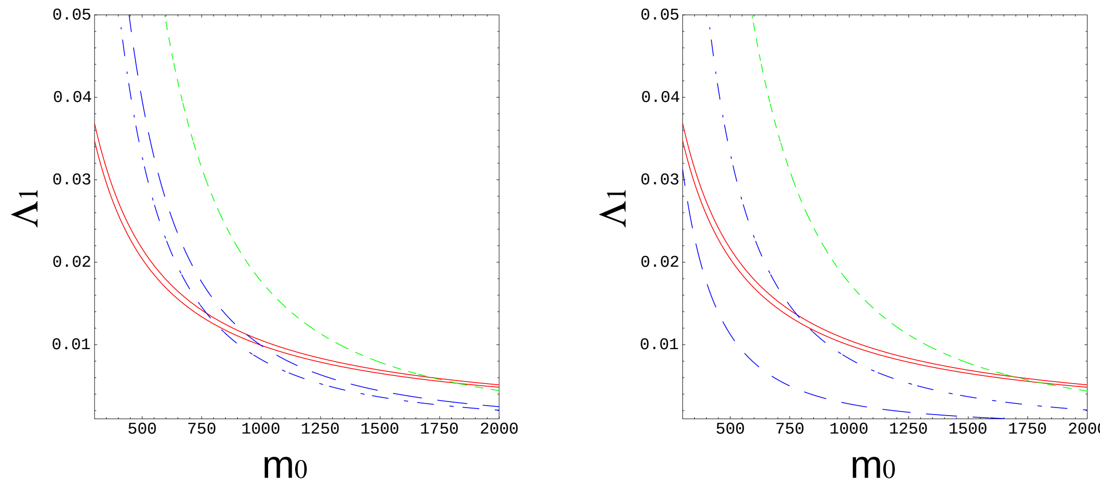

Fig. 1 Regions sandwiched by red solid lines satisfy the WMAP data $\Omega h^2 = 0.11 \pm 0.06$ for CDM abundance. Blue dashed and blue dash-dotted lines show the bounds for $\mu \to e\gamma$ and $\tau \to \mu\gamma$, respectively. Green dotted lines represent contours for $\delta a_\mu = 1.0 \times 10^{-11}$. The values of r_1 and r_3 are fixed as $r_1 = 0.8$ in both graphs and $r_3 = 4$ and 10 in the left and right graphs.

However, if we take account of the assumed conditions (8) in our model, we find that s-wave contributions cancel out and only p-wave contributions remain. Thus, the relevant cross section is found to be written as

$$\sigma_{N_{i_1}N_{i_2}}|v_{\rm rel}| = \frac{1}{3\pi}\frac{M_1^2(M_1^4+m_0^4)}{(M_1^2+m_0^2)^4}h_{\tau i_1}^2 h_{\tau i_2}^2 v_{\rm rel}^2. \tag{19}$$

We have $a_{\rm eff} = 0$ and

$$b_{\rm eff} = \frac{1}{12\pi m_0^2}\frac{r_1^2(1+r_1^4)}{(1+r_1^2)^4}(h_{\tau 1}^2 + h_{\tau 2}^2)^2, \tag{20}$$

where we define r_1 by $r_1 = M_1/m_0$. Applying this effective cross section to eq. (17), we can estimate the relic abundance of N_1.

In the case (i), the constraints (11) obtained from the neutrino oscillation data are written as

$$h_{\tau 3}^2\Lambda_3 \simeq 2.9 \times 10^{-3}~{\rm eV}, \qquad (h_{\tau 1}^2 + h_{\tau 2}^2)\Lambda_1 \simeq 2.5 \times 10^{-2}~{\rm eV}. \tag{21}$$

If we use this relation in eq. (20), the effective annihilation cross section of N_1 can be expressed by using Λ_1 instead of Yukawa couplings $h_{\tau 1}$ and $h_{\tau 2}$. As found from eq. (14), the bound of $\mu \to e\gamma$ directly constrains only $h_{\tau 3}$ which is not relevant to the N_1 annihilation in this case. Since the relevant Yukawa couplings $h_{\tau 1}$ and $h_{\tau 2}$ are constrained by the bound of $\tau \to \mu\gamma$, this bound can be much more severe constraint than that of $\mu \to e\gamma$.

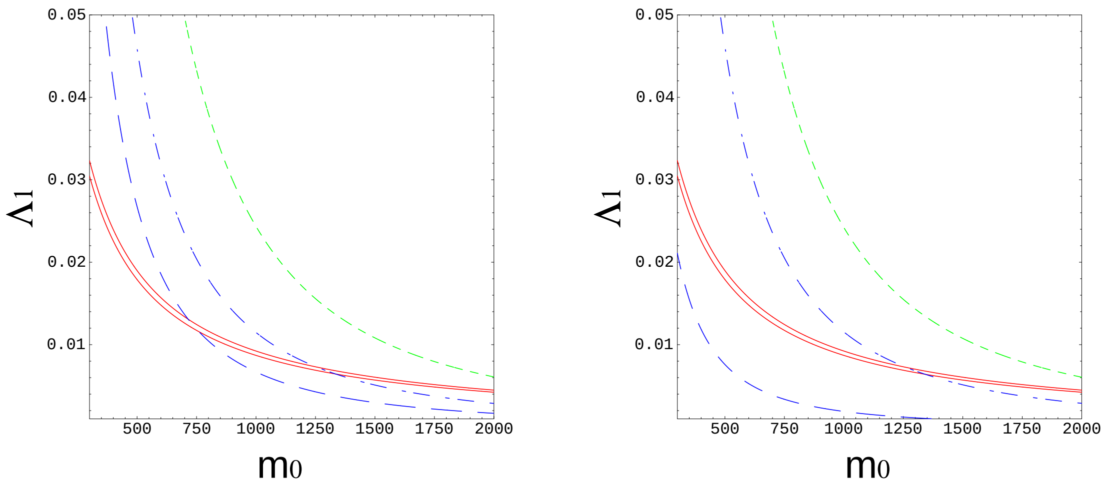

Fig. 2 The same figures as Fig. 1 for a different r_1. Although r_1 is fixed as $r_1 = 0.4$ in both graphs, r_3 is put as 4 and 10 in the left and right graphs as in Fig. 1.

These constraints can also be written as conditions on Λ_1 as follows,

$$\Lambda_1 > 3.0\, \frac{r_3 I(r_1)}{r_1 I(r_3)} F_2(r_3) \left(\frac{500\ \mathrm{GeV}}{m_0}\right)^2\ \mathrm{eV},$$
$$\Lambda_1 > \left(0.34 F_2(r_1) - 4.0 \times 10^{-2} \frac{r_3 I(r_1)}{r_1 I(r_3)} F_2(r_3)\right) \left(\frac{500\ \mathrm{GeV}}{m_0}\right)^2\ \mathrm{eV}, \qquad (22)$$

where we define r_3 by $r_3 = M_3/m_0$. We use a relation $\Lambda_3 = (r_1 I(r_3)/r_3 I(r_1))\Lambda_1$ obtained from eq. (4) and also the conditions in eq. (21) in this derivation.

Using these results, we plot favorable regions in the (m_0, Λ_1) plane by fixing the values of r_1 and r_3. Since our considering case (i) corresponds to $r_1 < 1$ and $r_3 \geq 1$, we fix these to some typical values. We plot examples of the allowed regions for $(r_1, r_3) = (0.8, 4)$ and $(0.8, 10)$ in Fig. 1 and also for $(r_1, r_3) = (0.4, 4)$ and $(0.4, 10)$ in Fig. 2. In both figures, blue dashed and blue dash-dotted lines show the bounds for $\mu \to e\gamma$ and $\tau \to \mu\gamma$, respectively. The upper regions of both lines satisfy these constraints. Thin bands sandwiched by the red solid lines corresponds to region to realize the value of Ωh^2 required by the WMAP data. Form this figure we find that the relic abundance of N_1 can be consistent with the WMAP data without conflicting the bounds of lepton flavor violating processes. Figs. 1 and 2 show that mass of N_1 should be larger than 700 GeV and 500 GeV for each case. Λ_3/Λ_1 takes values of $O(1)$, for example, 1.14 and 0.80 for $r_3 = 4$ and 10 in case of $r_1 = 0.8$, respectively. Although Λ_1 has rather small values ~ 0.01 in the allowed regions, Yukawa couplings $h_{\tau 1}$ and $h_{\tau 2}$ can be confirmed to be in the perturbative regions

by taking account of eq. (21). Since these couplings contribute to $\tau \to \mu\gamma$, this constraint can be much stronger than $\mu \to e\gamma$ as found from Figs. 1 and 2. However, we can find consistent solutions by fixing r_1 and r_3 suitably.

The reconciliation between the CDM abundance and the lepton flavor violating neutral processes can be shown to be accomplished even in the original radiative seesaw model without substantial modification of the model. In the present flavor structure of Yukawa couplings, the (co)annihilation of N_1 and $\mu \to e\gamma$ are induced by the different ones, respectively. Yukawa couplings relevant to the (co)annihilation of N_1 contribute to $\tau \to \mu\gamma$ whose bound is much weaker than $\mu \to e\gamma$. This feature makes their reconciliation possible by arranging the masses of singlet fermions so as to satisfy the requirements from the neutrino oscillation data.

Finally we give remarks on some predictions of the model. Both direct and indirect detections of dark matter are crucial to judge whether the considering model for dark matter is viable or not [18, 19]. Since the CDM candidate has couplings only to μ and τ, it can decay to these. Model independent analysis of the data of PAMELA and ATIC/PPB-BETS has been done in [20, 21]. Ref.[21] suggests that the best fit is obtained for $M \sim 1$ TeV with CDM annihilating into $\mu^+\mu^-$ and a good fit is obtained for $M \sim 2$ TeV with CDM annihilating into $\tau^+\tau^-$. It is interesting that this is consistent with our results obtained in the present analysis. Encouraged by this result, we would like to add some qualitative arguments on the related subjects in our particular model.

As shown in eq. (19), N_1 annihilation cross section $\sigma|v|$ is dominated by p-wave contribution due to helicity suppression. However, although p-wave contribution which has v^2 dependence dominates the annihilation cross section at freeze out time where $v \sim 0.2$, it is largely suppressed in the present Galaxy where $v \sim 10^{-3}$. This makes s-wave contribution relevant to PAMELA anomaly rather than the p-wave contribution. Since s-wave annihilation cross section can be estimated as $\sigma|v| \simeq \frac{h_{\alpha 1}^2}{8\pi}\frac{m_f^2}{m_0^4(1+r_1^2)^2}$ where m_f is the mass of final fermions, N_1 annihilation in the Galaxy occurs mainly through $N_1N_1 \to \tau^+\tau^-$. If we use typical values of m_0, $h_{\tau 1}$ and r_1 obtained as the solutions consistent with the WMAP data in this paper, we find that the boost factor should be $O(10^6)$ or larger.[††] The model cannot induce this amount of enhancement for the annihilation cross section in the present form. For the explanation of this boost factor, there may be two possibilities: (i)

[††]We note that $\sigma|v| \sim 10^{-23}$ cm^3/sec is required to explain the positron excess in the PAMELA data.

the model should be extended such that the relic N_1 has a large non-thermal component as discussed in [22], for example, or (ii) it should be explained by some astrophysical effects. However, it seems difficult to obtain substantial effects by a simple extension referred in case (i). Since we can not make the annihilation cross section of N_1 itself larger preserving the features of the model, main effect should come from the increase of number density of the non-thermal component of relic N_1 as a result of the decay of other fields. However, it is severely constrained by the WMAP data and we have no freedom to obtain the large boost factor mentioned above.

Bremsstrahlung from the charged fields associated to this annihilation $N_1N_1 \to \tau^+\tau^-$ yields diffuse photons. We may check the model by comparing the flux of diffuse photon expected from this N_1 annihilation based on the PAMELA data with observations such as Hess and Fermi/GLAST. Such a model independent analysis is presented in [23]. Although it suggests that dark matter annihilation with $\tau^+\tau^-$ final states may be difficult to be consistent with diffuse photon data, the assumptions in that analysis seems not to be applied to our model. On the other hand, dark matter annihilation into final states composed of three fields such as $e^+e^-\gamma$ can be dominant processes if the annihilation cross section is helicity suppressed [24]. The radio emission from synchrotron radiation and γ-ray emission from inverse Compton scattering from the charged fields produced by the dark matter annihilation may be also useful to discriminate the origin of positron excess [25]. Observational data of diffuse photon obtained in the Fermi/GLAST experiment may give us crucial hints for these [26]. Anyway, detailed analysis of diffuse photon is necessary to check the validity of the present model.

Decay of $\tau^\pm$ also produces neutrino flux. If we use the PAMELA positron data, its flux can be roughly estimated as $O(10^{5\sim 6})$ GeV/(cm^2·sec·str) at the relevant neutrino energy. This flux is larger than neutrino flux expected from certain types of AGN but smaller than the atmospheric neutrino flux [27]. This suggests that the N_1 annihilation is difficult to be detected through the observation of the neutrino flux on the Earth.

Direct detection of N_1 is also an interesting subject. Although η has the interaction shown in eq. (1) and no direct interaction with quarks, N_1 can be scattered by nuclei through one-loop effect with Z boson exchange. Since N_1 is a Majorana fermion, this effective interaction with quark is expressed by an axial vector interaction $d_q\bar{N}_1\gamma_5\gamma_\mu N_1\bar{q}\gamma_5\gamma^\mu q$ with $d_q \sim \frac{g_2^2h_{\tau 1}^2r_1^2}{(4\pi)^2m_W^2}T_{3q}$, which yields spin dependent scattering. If we use the parame-

ters obtained in this paper, this spin dependent cross section is roughly estimated as $O(10^{-41})$ cm^2. This is much smaller than the present bound of spin dependent elastic scattering cross section for dark matter with $O(1)$ TeV mass [28]. Thus, it seems difficult to find this dark matter even in the next generation direct detection experiments. Studies related to these aspects of the similar model can also be found in [29] although lepton number violating constraints are not taken into account there.

In addition to these indirect and direct search of dark matter, there may be some other phenomena which could show characteristic features of the model. The effective mass in the neutrinoless double β decay is given as a fixed value $m_{\rm eff} = \sqrt{\Delta m^2_{\rm sol}}\sin\theta_{\rm sol} \simeq 2.9\times 10^{-3}$ eV, which is one order of magnitude below the reach of near future experiments. Present data for the magnetic dipole moment of muon shows discrepancy between a value predicted by the SM and the experimental result [30]. In the present model there is one-loop contribution to δa_μ, which is estimated as [11]

$$\begin{aligned} \delta a_\mu &= \sum_{k=1}^{3} \frac{h_{\mu k}^2}{(4\pi)^2}\frac{m_\mu^2}{m_0^2}F_2(r_k) \\ &\simeq \frac{7.1\times 10^{-12}}{\Lambda_1}\left(\frac{500\ GeV}{m_0}\right)^2\left[F_2(r_1) + 0.12\frac{r_3 I(r_1)}{r_1 I(r_3)}F_2(r_3)\right]. \end{aligned} \tag{23}$$

A contour for $\delta a_\mu = 1.0\times 10^{-11}$ is also plotted by a green dotted line in the figures. This shows that these values predicted by our model are two orders of magnitude smaller than $\delta a_\mu = (30.2\pm 8.7)\times 10^{-10}$ [31]. In order to improve this situation for the δa_μ, additional contributions to δa_μ are required in our model. Such contributions may be obtained by embedding our scenario in supersymmetric models, in which an ordinary supersymmetric CDM candidate such as the lightest neutralino does not dominate the required relic abundance. We will discuss such extensions elsewhere.

4 Summary

The radiative seesaw model considered in this paper is one of interesting possibilities to explain the origin of neutrino masses. It can include a cold dark matter candidate as an important ingredient of the neutrino mass generation. However, the model has been considered to have a severe discrepancy between magnitude of Yukawa couplings required by the dark matter relic abundance and the suppression of lepton flavor violating neutral

processes. In this study we have proposed a new possibility to relax this tension within the original minimal radiative seesaw model without introducing additional interactions. We have found that the model can overcome this problem simultaneously satisfying the conditions required by the neutrino oscillation data as long as the Yukawa couplings and also the mass hierarchy of the singlet fermions have appropriate structure. The present study shows that even the minimal radiative seesaw model can be an interesting candidate for models which relates neutrino masses to the existence of dark matter.

The model may be relevant to the PAMELA e^+ and $\bar{p}$ data, and also the ATIC/PPB-BETS $(e^+ + e^-)$ data. We have briefly presented qualitative observations on the detection of the diffuse particles produced in the dark matter annihilation and the direct search of this dark matter. Since these analyses are rough and qualitative ones, we need more quantitative study to mention on the predictions of the model in detail. This scenario may play an important role in some supersymmetric models if it is embedded in the supersymmetric framework. Although we have considered only the model with restricted coannihilation processes here, other cases are also expected to give interesting possibilities. These points may be worthy for further study and will be discussed elsewhere.

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